Documentos CEDE

ISSN 1657-7191 edición electrónica

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29

OCTUBRE DE 2010

Universidad de los Andes
Facultad de Economía

CEDE
Centro de Estudios sobre Desarrollo Económico

Serie Documentos Cede, 2010-29
ISSN 1657-7191

Octubre de 2010

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Edición, diseño de cubierta, preprensa y prensa digital:
Proceditor ltda.
Calle 1ª C No. 27 A – 01
Bogotá, D. C., Colombia
Teléfonos: 2204275, 220 4276, Fax: extensión 102
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Impreso en Colombia – *Printed in Colombia*

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Abstract

This paper theoretically assesses the role that uncertainty plays in the intensity of conflicts. The standard two-player rent-seeking contest model (Tullock, 1980) is extended to allow for privately known subjective values of the prize. The conflict is modeled as a Bayesian game on which each player's valuation is drawn independently from arbitrary distributions. We find sufficient conditions for when first-order and second-order stochastic refinements in the distributions cause predictable movements in the conflict's dissipation. We focus on arbitrary contest success functions and arbitrary independent distributions for each player, allowing us to extend our analysis beyond the case of symmetric equilibria.

Keywords: conflict, uncertainty, monotone comparative statics, Bayesian games.

JEL Classification Numbers: C70, C72, D74, D80.

*The views expressed in the paper are those of the author and do not represent those of Banco de la República or its Board of Directors. All errors and omissions are mine.

Incertidumbre en Conflictos

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Resumen

Este artículo estudia el papel que la incertidumbre juega en la determinación de la intensidad de un conflicto. El modelo estándar de búsqueda de rentas (Tullock, 1980) es extendido para permitir valoraciones privadas del premio. El conflicto es modelado como un juego Bayesiano en el que la valoración de cada jugador es tomada de distribuciones arbitrarias e independientes. Encontramos condiciones suficientes para cuando refinamientos estocásticos de primer y segundo orden en las distribuciones causan movimientos predecibles en la disipación del conflicto. Nos enfocamos en funciones de éxito arbitrarias e independientes para cada jugador, permitiéndonos extender el análisis más allá del caso de equilibrios simétricos.

Palabras Clave: conflicto, incertidumbre, estática comparativa, juegos Bayesianos.

Clasificación JEL: C70, C72, D74, D80.

*Los resultados y opiniones presentadas en este trabajo son responsabilidad exclusiva del autor y su contenido no compromete al Banco de la República ni a su Junta Directiva.

1 Introduction

Following Esteban and Ray (1999), a conflict is "a situation in which, in the absence of a collective decision rule, social groups with opposed interests incur losses in order to increase the likelihood of obtaining their preferred outcome". The dissipation of the conflict is defined as the sum of those losses, and this measure reflects the intensity of the conflict. These definitions apply to a rather large set of situations: military wars, commercial struggles, political campaigns, legal battles, auctions etc.

In a conflict there is uncertainty because of the incomplete information, when an agent does not know some characteristics of the opposite agents, or when the agents are not sure about the path followed by exogenous variables, such as random geographic or climatic events. This lack of information affects the optimal choices of the agents, making them to react to expected values formed with limited information sets.

The relative strength or weakness of the opponent perceived by the agents depends on what they believe about the unknown factors of the conflict (resources, technology, preferences etc.) and this determines the optimal decisions taken. Changes in what agents believe generate changes in the resources devoted to the conflict, and therefore in its intensity.

When discussing the importance of perceptions in the conflict, Hirshleifer (1993) wonders about the relation of these perceptions with conflict's intensity. According to him, some authors as Blainey (1973) argue that by knowing the real opportunities the intensity of the conflict decays. However Wittman (1979) argues that the certainty about the weakness of one side makes the strong party more aggressive. We will illustrate that both views can be valid within the neoclassical framework where the conflict is modeled as a Bayesian game.

Following the conflict theory literature, the standard two-player rent-seeking contest model (Tullock (1980)) is extended, allowing for privately known subjective values for the prize. This introduces incomplete information and allows us to study the effects of uncertainty on the optimal decisions of the agents. First we analyze a simple sequential contest in which an uninformed agent has to play first. Then we develop a simultaneous two-player rent-seeking contest on which each agent has private information about her subjective valuation for the prize in dispute. This is modeled as a Bayesian game on which each player's valuation is drawn independently from arbitrary distributions. In particular we find sufficient conditions on the contest success functions for when first-order and second-order stochastic refinements in the Bayesian priors cause predictable movements in the conflict's dissipation. Since these stochastic changes reflect somehow a higher level of uncertainty, we seek to find under what conditions a higher level of uncertainty generates a more intense conflict.

We focus on arbitrary contest success functions and arbitrary independent distributions for each player, allowing us to extend our analysis beyond the case of symmetric equilibria. This sets this paper apart from other that consider information in conflicts. For example, Hopkins and Kornienko (2004) analyze the monotone comparative statics induced by stochastic changes in a symmetric game, Malueg and Yates (2004) present models with two-sided incomplete information with Bernoulli distributions, Fey (2008) analyzes symmetric equilibria with a uniform distribution of costs and Ewerhart (2010) considers symmetric rent-seeking contests with independent private valuations of the contest prize and a particular two-parameter distribution. Finally, Wärneryd (2003) analyzes the ex-ante expected dissipation of contests with symmetric and asymmetric information and common value of the prize. We extend his results allowing for stochastic changes starting from any distribution, not just starting from the symmetric information case.

The paper is organized as follows. Section 2 introduces some basic mathematical definitions and results on monotone comparative statics and stochastic ordering. Section 3 presents the model for a sequential conflict. Section 4 extends the analysis to a simultaneous game of conflict, and specializes for the case of asymmetric information and common valuation. Section 5 provides some examples using standard functional forms. Section 6 offers some concluding remarks.

2 Monotone comparative statics and stochastic ordering

Let $f : \mathbb{R}^2 \to \mathbb{R}$. Consider the optimization problem

$$\max_{x \in X} \qquad f(x,t)$$

where $X \subseteq \mathbb{R}$ is the choice set, and $t \in T \subseteq \mathbb{R}$ is a parameter set. Let

$$X^*(t) \equiv \arg\max_{x \in X} f(x,t)$$

be the solution correspondence and $x^*(t) \in X^*(t)$ be a maximizer. This set changes when the parameter t changes. We are interested in analyzing how changes in t change the optimal behaviour of optimizing agents. Results of this kind are called "monotone comparative statics results".

If $X^*(t)$ is a set, we need to define an ordering of sets to answer this question.

Definition 1. Strong set order in $\mathbb{R}$.

Let A and B be to real sets. We say that the set A is greater than or equal to the set B in the strong set order, denoted as $A \geq_S B$, if and only if for all $a \in A$, $b \in B$ we have

$$\min\{a, b\} \in B \qquad \max\{a, b\} \in A$$

This implies that if $a \in A$, $b \in B$ and $b \geq a$ we must have $a \in B$ and $b \in A$.

Note that if $A = \{a\}$ and $B = \{b\}$ are singleton sets, then $A \geq_S B$ if and only if $a \geq b$.

We state the conditions on f that are sufficient to obtain monotone comparative statics results.

Definition 2. Increasing differences and single crossing.

Let $f : X \times T \to \mathbb{R}$ where $X \subseteq \mathbb{R}$, $T \subseteq \mathbb{R}$.

- f has increasing differences in (x, t) if and only if for all $x' > x$, $t' > t$ we have

$$f\left(x', t'\right) - f\left(x, t'\right) \geq f\left(x', t\right) - f(x, t)$$

- f satisfies the single crossing property in (x, t) if and only if for all $x' > x$, $t' > t$ we have

$$f\left(x', t\right) \geq (>)\, f(x, t) \Longrightarrow f\left(x', t'\right) \geq (>)\, f\left(x, t'\right)$$

Note that increasing differences implies the single crossing property.

Topkis' theorem will give us the monotone comparative statics result we need if f has increasing differences.

Theorem 1. *Topkis' univariate monotonicity theorem.*

Let $X \subseteq \mathbb{R}$, $T \subseteq \mathbb{R}$. Let $f : X \times T \to \mathbb{R}$ have increasing differences in (x, t). If $t' \geq t$ then $X^*(t') \geq_S X^*(t)$.

Proof. Topkis (1968, 1998). □

The following Theorem, due to Milgrom and Shannon (1994), will give us the monotone comparative statics result we need if f satisfies the single crossing property.

Theorem 2. *Milgrom-Shannon.*

Let $f : X \times T \to \mathbb{R}$ satisfy the single crossing property in (x, t). If $t' \geq t$ then $X^* (t') \geq_S X^* (t)$.

Proof. Milgrom and Shannon (1994). $\square$

The purpose of this paper is to analyze the role of uncertainty in the equilibrium outcomes of a conflict. To obtain monotone comparative statics results regarding the uncertainty faced by agents, we need some concept of ordering of probability distributions.

Definition 3. First-order stochastic dominance

Let $X = [a, b] \subseteq \mathbb{R}$, where $-\infty < a < b < \infty$. Let $F(x)$ and $G(x)$ be two distributions defined on X.

$F(x)$ first-order stochastically dominates $G(x)$ if for every non-decreasing function $f : X \to \mathbb{R}$ we have

$$\int_a^b f(x)\, dF(x) \geq \int_a^b f(x)\, dG(x)$$

Let $x \in X$. If we define $f(t) = \begin{cases} 0 & \text{if } t \leq x \\ 1 & \text{if } x < t \end{cases}$, then $\int_a^b f(t)\, dF(t) = \int_x^b dF(t) = 1 - F(x)$. If $F(x)$ first-order stochastically dominates $G(x)$ then we conclude, for all $x \in X$,

$$F(x) \leq G(x)$$

It turns out that this property is equivalent to the first-order stochastic dominance. The intuition then is that it is more likely to realize higher outcomes with $F(x)$ than with $G(x)$.

Definition 4. Second-order stochastic dominance and mean-preserving spread.

Let $X = [a, b] \subseteq \mathbb{R}$, where $-\infty < a < b < \infty$. Let $F(x)$ and $G(x)$ be two distributions defined on X with the same mean.

- $F(x)$ second-order stochastically dominates $G(x)$ if for every non-decreasing concave function $f : X \to \mathbb{R}$ we have

$$\int_a^b f(x)\, dF(x) \geq \int_a^b f(x)\, dG(x)$$

- Let X_F and X_G be random variables associated with the distributions $F(x)$ and $G(x)$. Then $G(\cdot)$ is a mean-preserving spread of $F(\cdot)$ if and only if $X_G \overset{d}{=} X_F + Z$ for some random variable Z having $E(Z|X_F) = 0$ for all values of X_F.

These two definitions are equivalent (see Rothschild and Stiglitz (1970) and Mas-Colell, Whinston, and Green (1995, Section 6.D) for example).

Thus, if $F(x)$ second-order stochastically dominates $G(x)$, we have that G has more variance than F. This fact also follows from the following result.

Theorem 3. *If $F(x)$ second-order stochastically dominates $G(x)$ then $\int_a^b f(x)\,dF(x) \geq \int_a^b f(x)\,dG(x)$ for all concave function f (notice that f does not have to be non-decreasing).*

Proof. Rothschild and Stiglitz (1970). □

3 A sequential game of conflict

Assume that two groups are competing for the appropriation of a valuable resource (for instance, ruling a country, controlling a population or a key resource etc.). Each group assigns an exogenous subjective value to the resource, and this value is drawn from some distribution. Then they choose a level of effort to increase the likelihood of obtaining the preferred result, subject to a resource constraint and a typical contest success function[1]. This level of effort can be seen as the resources that each agent dedicates to the conflict with the goal of obtaining the disputed good.

In the sequential game, agent j chooses first, after observing the realization of her subjective valuation, but not knowing the valuation of the opponent. Then agent i chooses her effort level, having observed the choices of agent j.

This model applies to several sequential conflicts in which some agent has to publicly announce and commit to her strategy. For example, agent j can be a country's government, required by law to reveal to the public the amount of resources used in the conflict against some terrorist group. Government however does not know the real valuation given by agent i to the spoils of conflict, and therefore does not know their real strength. On the other hand, agent i can be a terrorist group fighting against the government and not subject to the law. They have the advantage to know beforehand how many resources government is spending in the fight.

[1] The contest success function represents the technology of the conflict. Some functional forms and their properties are studied by Hirshleifer (1989).

But of course conflict does not imply violence. We could let agent j to be the prosecution team, responsible for presenting the case in a criminal trial. Once evidence has been revealed and prosecution's strategy is known, the defense team acts. That is, agent i plays. However it could not be clear to the prosecutor the strength of the defense.

Formally, we assume that the agents are risk-neutral, and their expected utility is given by

$$v_i p_i(e_i, e_j) - e_i$$

where v_i is the subjective valuation for the good in dispute of agent i, e_i is the effort level of agent i, e_i is the effort level of agent i and $p_i(e_i, e_j)$ is a standard contest success function (CSF). This function satisfies

$$\begin{aligned} &0 \leq p_i(e_i, e_j) \leq 1 \\ &\frac{\partial}{\partial e_i} p_i(e_i, e_j) > 0 \qquad \frac{\partial}{\partial e_j} p_i(e_i, e_j) < 0 \end{aligned}$$

for all $e_i, e_j \geq 0$. For given effort levels (e_i, e_j), $p_i(e_i, e_j)$ assigns a probability for agent i to get the good in dispute. This probability is increasing in own effort and decreasing in the effort of the opponent. Of course agent j will also have a contest success function given by

$$p_j(e_j, e_i) = 1 - p_i(e_i, e_j)$$

where we assume that the good in dispute is not wasted.

Uncertainty is introduced in this model assuming that the groups in the conflict do not know the real nature of their opponents: in particular, the subjective valuation assigned to the disputed good by the opponent is unknown. This is a game with incomplete information.

Nature chooses v_i and v_j independently from the distributions $F_i(v_i)$ and $F_j(v_j)$, with support $v_i \in [\underline{v}_i, \overline{v}_i]$ and $v_j \in [\underline{v}_j, \overline{v}_j]$, where $0 < \underline{v}_i < \overline{v}_i < \infty$ and $0 < \underline{v}_j < \overline{v}_j < \infty$. This determines the type of the agents. This way of modelling the uncertainty is consistent with the example proposed by Hirshleifer (1993): a higher subjective valuation of the good in dispute may be understood as a higher desire to fight for it, and can be associated with a stronger opponent.

Given the valuations, agent j plays first, but she does not know the realization of v_i. Then the choice of $e_j(v_j)$ is revealed to agent i, who chooses e_i with full information.

This is a Bayesian game, and we will use the subgame perfect Bayesian Nash equilibrium as solution concept. A strategy for agent j is a function $e_j : \left[\underline{v}_j, \overline{v}_j\right] \to \mathbb{R}_+$ such that for each possible type chooses a non-negative effort level. A strategy for agent i is a function $e_i : [\underline{v}_i, \overline{v}_i] \times \mathbb{R}_+ \to \mathbb{R}_+$ such that for each type and each announcement $e_j \geq 0$ chooses a non-negative effort level.

We will now characterize the equilibria. We solve this sequential game by backward induction.

Given a strategy $e_j(v_j)$ of agent j, the problem faced by agent i is:

$$\max_{e_i \geq 0} \qquad v_i p_i\left(e_i, e_j\left(v_j\right)\right) - e_i$$

This maximization problem always has a solution. Note that for any $e_j(v_j) \geq 0$ the best response will satisfy $e_i^* \in [0, v_i]$. If $e_i^* > v_i$ then $v_i p_i\left(e_i^*, e_j\left(v_j\right)\right) - e_i^* < 0$ and this can be improved by setting $e_i = 0$. Then agent i is optimizing a continuous function in a compact set.

The solution can be characterized by the Kuhn-Tucker condition

$$\begin{aligned} v_i \frac{\partial}{\partial e_i} p_i\left(e_i^*, e_j\left(v_j\right)\right) - 1 = 0 \qquad & \text{for } e_i^* > 0 \\ v_i \frac{\partial}{\partial e_i} p_i\left(e_i^*, e_j\left(v_j\right)\right) - 1 \leq 0 \qquad & \text{if } e_i^* = 0 \end{aligned}$$

and this defines the implicit best-response function of agent i to strategy $e_j(v_j)$.

A solution $e_i^*(v_i, e_j)$ that satisfies the first-order condition with equality for all relevant values of e_j will be called an "interior solution".

We could impose some assumptions to get interior solution. Assume that $\frac{\partial}{\partial e_i} p_i(0, e_j)$ is decreasing. Since $\frac{\partial}{\partial e_i} p_i(0, e_j) > 0$ we have that there exists a unique $\underline{v}_i(e_j)$ such that

$$\underline{v}_i\left(e_j\right) \frac{\partial}{\partial e_i} p_i\left(0, e_j\right) = 1$$

It is easy to see that $\underline{v}_i(e_j)$ is a non-decreasing function. In the differentiable case and under our assumptions

$$\frac{d}{de_j} \underline{v}_i\left(e_j\right) = -\frac{\frac{\partial^2}{\partial e_j \partial e_i} p_i\left(0, e_j\right)}{\frac{\partial}{\partial e_i} p_i\left(0, e_j\right)} > 0$$

With this definition we have that the best response is $e_i^*(v_i, e_j) = 0$ if $\underline{v}_i \leq v_i < \underline{v}_i(e_j)$

and satisfies the first-order condition with equality for $v_i \geq \underline{v}_i(e_j)$. If we assume that $v_i \geq \underline{v}_i(v_j)$ then we have a interior solution for all possible equilibrium values of e_j.

Under some assumptions we can show the best-response function to be increasing, and for interior solution, concave in v_i.

Proposition 1. *Assume the following:*

- *The optimization problem has a unique interior solution,*
- $\frac{\partial}{\partial e_i} p_i(e_i, e_j) > 0$ *for all* e_i, e_j,
- $\frac{\partial^2}{\partial e_i^2} p_i(e_i^*(v_i, e_j), e_j) < 0$ *(second-order necessary condition for maximization),*
- *and* $\frac{\partial^3}{\partial e_i^3} p_i(\cdot) \frac{\partial}{\partial e_i} p_i(\cdot) \leq 2\left(\frac{\partial^2}{\partial e_i^2} p_i(\cdot)\right)^2$ *(this is satisfied if* $\frac{\partial^3}{\partial e_i^3} p_i(e_i, e_j) \leq 0$*).*

Then the best-response function to the strategy $e_j(v_j)$ *is an increasing and concave function of* v_i.

We can prove the fact that the best-response correspondence is non-decreasing in the strong set order (without requiring interior solution) just by noting that

$$\frac{\partial^2}{\partial v_i \partial e_i} \Pi_i(e_j(v_j)) = \frac{\partial}{\partial e_i} p_i(e_i, e_j(v_j)) > 0$$

This implies that the objective function has increasing differences in (e_i, v_i) and by Topkis' Theorem the result follows. However, we prefer to abuse the power of calculus to obtain stronger conclusions.

Proof. We have that the best-response function $e_i^*(v_i)$ satisfies the first-order condition

$$v_i \frac{\partial}{\partial e_i} p_i(e_i^*(v_i, e_j), e_j) = 1$$

Applying implicit differentiation we get

$$\frac{\partial}{\partial v_i} e_i^*(v_i, e_j) = -\frac{\frac{\partial}{\partial e_i} p_i(e_i^*(v_i, e_j), e_j)}{v_i \frac{\partial^2}{\partial e_i^2} p_i(e_i^*(v_i, e_j), e_j)} > 0$$

because $\frac{\partial}{\partial e_i} p_i(e_i, e_j) > 0$ and $\frac{\partial^2}{\partial e_i^2} p_i(e_i^*(v_i, e_j), e_j) < 0$ by the second-order necessary condition for the maximization of the objective function.

Taking derivatives we get

$$\frac{\partial^2}{\partial v_i^2} e_i^*\left(v_i, e_j\right) = -\frac{v_i\left(\frac{\partial^2}{\partial e_i^2} p_i\left(\cdot\right)\right)^2 \times \frac{\partial e_i^*}{\partial v_i} - \frac{\partial}{\partial e_i} p_i\left(\cdot\right) \times \left(\frac{\partial^2}{\partial e_i^2} p_i\left(\cdot\right) + v_i \frac{\partial^3}{\partial e_i^3} p_i\left(\cdot\right) \times \frac{\partial e_i^*}{\partial v_i}\right)}{\left[v_i \frac{\partial^2}{\partial e_i^2} p_i\left(\cdot\right)\right]^2}$$

and substituting for $\frac{\partial e_i^*(v_i)}{\partial v_i}$ we conclude that if $\frac{\partial}{\partial e_i} p_i\left(e_i, e_j\right) > 0$ and $\frac{\partial^2}{\partial e_i^2} p_i\left(e_i^*\left(v_i, e_j\right), e_j\right) < 0$ then

$$\frac{\partial^2 e_i^*\left(v_i\right)}{\partial v_i^2} \leq 0 \iff \frac{\partial^3}{\partial e_i^3} p_i\left(\cdot\right) \frac{\partial}{\partial e_i} p_i\left(\cdot\right) \leq 2\left(\frac{\partial^2}{\partial e_i^2} p_i\left(\cdot\right)\right)^2$$

where $p_i\left(\cdot\right)$ represents $p_i\left(e_i^*\left(v_i, e_j\right), e_j\right)$. $\square$

For the next stage of the backward induction we need to solve the optimization problem of agent j. Given a strategy $e_i\left(v_i, e_j\right)$ followed by agent i, agent j solves the problem

$$\max_{e_j \geq 0} \qquad v_j \int_{\left[\underline{v}_i, \overline{v}_i\right]} p_j\left(e_j, e_i\left(v_i, e_j\right)\right) dF_i - e_i$$

where $F_i\left(v_i\right)$ is the distribution from where v_i is drawn. This reflects the fact that v_i is unknown to agent j.

Consider two distributions $F_i\left(v_i\right)$ and $G_i\left(v_i\right)$. We want to compare the optimal response of agent j to a given strategy $e_i\left(v_i, e_j\right)$ of agent i under these two distributions.

Let $U_j\left(e_j, t; e_i\left(v_i, e_j\right)\right)$ be the parametrized objective function

$$U_j\left(e_j, t; e_i\left(v_i, e_j\right)\right) \equiv v_j\left(t \int_{\left[\underline{v}_i, \overline{v}_i\right]} p_j\left(e_j, e_i\left(v_i, e_j\right)\right) dF_i + (1-t) \int_{\left[\underline{v}_i, \overline{v}_i\right]} p_j\left(e_j, e_i\left(v_i, e_j\right)\right) dG_i\right) - e_j$$

for $v_j \in \left[\underline{v}_j, \overline{v}_j\right]$, $e_j \in \left[0, v_j\right]$ and parameter $t \in \{0, 1\}$.

We will use the single crossing property to get some monotone comparative statics results. The following proposition give sufficient conditions for the single crossing property to hold, under different assumptions on the distributions $F\left(x\right)$ and $G\left(x\right)$.

Proposition 2. *Fix a strategy $e_i\left(v_i, e_j\right)$ followed by agent i. For all $e_j' > e_j$, let $\Delta p_j\left(v_i\right) \equiv p_j\left(e_j', e_i\left(v_i, e_j'\right)\right) - p_j\left(e_j, e_i\left(v_i, e_j\right)\right)$.*

- *Assume that $\Delta p_j\left(v_i\right)$ is a non-increasing (non-decreasing) function. If G_i first-order* *stochastically dominates F_i, then the objective function $U_j\left(e_j, t; e_i\left(v_i, e_j\right)\right)$ ($U_j\left(e_j, 1-t; e_i\left(v_i, e_j\right)\right)$) satisfies the single crossing property in $\left(e_j, t\right)$.*

- *Assume that $\Delta p_j\left(v_i\right)$ is a convex (concave) function. If F_i is a mean-preserving spread of G_i, then the objective function $U_j\left(e_j, t; e_i\left(v_i, e_j\right)\right)$ ($U_j\left(e_j, 1-t; e_i\left(v_i, e_j\right)\right)$) satisfies the single crossing property in $\left(e_j, t\right)$.*

For the differentiable case, let $dp\left(e_j, v_i\right) = \frac{\partial}{\partial e_j} p_j\left(e_j, e_i\left(v_i, e_j\right)\right) + \frac{\partial}{\partial e_i} p_j\left(e_j, e_i\left(v_i, e_j\right)\right) \frac{\partial}{\partial e_j} e_i\left(v_i, e_j\right)$. The conclusions of Proposition 2 are satisfied if $dp\left(e_j, v_i\right)$ is non-increasing (non-decreasing) and/or convex (concave) in v_i.

Proof. Let $e_j' > e_j$. Note that

$$\begin{aligned} & U_j\left(e_j', 0; e_i\left(v_i, e_j'\right)\right) > (\geq) U_j\left(e_j, 0; e_i\left(v_i, e_j\right)\right) \\ \iff \quad & v_j \int_{[\underline{v}_i, \overline{v}_i]} \left(p_j\left(e_j', e_i\left(v_i, e_j'\right)\right) - p_j\left(e_j, e_i\left(v_i, e_j\right)\right)\right) dG_i > (\geq) e_j' - e_j \end{aligned}$$

Therefore, a sufficient condition for the single crossing property of $U_j\left(e_j, t; e_i\left(v_i, e_j\right)\right)$ is

$$\int_{\underline{v}_i}^{\overline{v}_i} \left(p_j\left(e_j, e_i\left(v_i, e_j\right)\right) - p_j\left(e_j', e_i\left(v_i, e_j'\right)\right)\right) dG_i \geq \int_{\underline{v}_i}^{\overline{v}_i} \left(p_j\left(e_j, e_i\left(v_i, e_j\right)\right) - p_j\left(e_j', e_i\left(v_i, e_j'\right)\right)\right) dF_i$$

If $p_j\left(e_j, e_i\left(v_i, e_j\right)\right) - p_j\left(e_j', e_i\left(v_i, e_j'\right)\right) = -\Delta p_j\left(v_i\right)$ is a constant function, this inequality is satisfied trivially.

Assume now that $-\Delta p_j\left(v_i\right)$ is not constant.

Let G_i first-order stochastically dominate F_i. Then, if $-\Delta p_j\left(v_i\right)$ is non-decreasing the inequality holds. For the differentiable case, a sufficient condition for single crossing is that

$$\frac{d}{de_j} p_j\left(e_j, e_i\left(v_i, e_j\right)\right) = \frac{\partial}{\partial e_j} p_j\left(e_j, e_i\left(v_i, e_j\right)\right) + \frac{\partial}{\partial e_i} p_j\left(e_j, e_i\left(v_i, e_j\right)\right) \frac{\partial}{\partial e_j} e_i\left(e_j, e_i\left(v_i, e_j\right)\right)$$

is non-increasing.

If G_i second-order stochastically dominates F_i, this inequality will hold if $-\Delta p_j\left(v_i\right)$ is a concave function of v_i (Theorem 3). For the differentiable case, a sufficient condition for single crossing is that $\frac{d}{de_j} p_j\left(e_j, e_i\left(v_i, e_j\right)\right)$ is a convex function of v_i.

Similar arguments hold if the objective function is $U_j\left(e_j, 1-t; e_i\left(v_i, e_j\right)\right)$. $\square$

Note that if the strategy followed by agent i is the optimal interior solution, then $e_i(v_i, e_j)$ satisfies the first-order condition $v_i \frac{\partial}{\partial e_i} p_i(e_i, e_j) = 1$ and we have

$$\frac{\partial e_i}{\partial e_j} = -\frac{\frac{\partial^2}{\partial e_j \partial e_i} p_i(e_i, e_j)}{\frac{\partial^2}{\partial e_i^2} p_i(e_i, e_j)}$$

Substituting this in the differentiable condition of Proposition 2 we get the following result.

Corollary 1. *If $e_i(v_i, e_j)$ is the interior solution of agent i's problem and it is non-decreasing (non-decreasing and concave) with respect to v_i, if the function $\frac{\partial}{\partial e_j} p_j(e_j, e_i) - \frac{\partial}{\partial e_i} p_j(e_j, e_i) \frac{\frac{\partial^2}{\partial e_i \partial e_j} p_j(e_j, e_i)}{\frac{\partial^2}{\partial e_i^2} p_j(e_j, e_i)}$ is non-increasing (non-increasing and convex) in e_i, and if G_i first-order (second-order) stochastically dominates F_i, then the objective function $U_j(e_j, t; e_i(v_i, e_j))$ satisfies the single crossing property in (e_j, t).*

Now we need to relate the changes in e_j with changes in e_i, in order to study the equilibrium dissipation level. This is achieved with the following result.|

Proposition 3. *Let $e_i^*(v_i, e_j)$ be the unique solution to agent i's problem. If one of the following holds, then $e_i^*(v_i, e_j)$ is non-decreasing in e_j:*

- *$p_i(e_i, e_j)$ has increasing differences in (e_i, e_j). That is, for $e_i' > e_i$, $e_j' > e_j$ we have*

  $$p_i(e_i', e_j') - p_i(e_i, e_j') \geq p_i(e_i', e_j) - p_i(e_i, e_j)$$

  Note that this holds, for the differentiable case, if $\frac{\partial^2}{\partial e_j \partial e_i} p_i(e_i, e_j) \geq 0$ for all e_i, e_j. This is a global condition.

- *We have interior solution and*

  $$\frac{\frac{\partial^2}{\partial e_j \partial e_i} p_i(e_i^*(v_i, e_j), e_j)}{\frac{\partial^2}{\partial e_i^2} p_i(e_i^*(v_i, e_j), e_j)} \leq 0$$

  This is a local condition.

Proof. The global result will follow if the objective function of agent i satisfies increasing differences in (e_i, e_j). Then, by Topkis' monotonicity theorem, $e_i^*\left(v_i, e_j'\right)$ is greater than or equal to $e_i^*(v_i, e_j)$ in the strong set order, for $e_j' \geq e_j$.

Let $e_i' > e_i$ and $e_j' > e_j$. Then

$$
\begin{aligned}
& \left(v_i p_i\left(e_i', e_j'\right) - e_i'\right) - \left(v_i p_i\left(e_i, e_j'\right) - e_i\right) \geq \left(v_i p_i\left(e_i', e_j\right) - e_i'\right) - \left(v_i p_i\left(e_i, e_j\right) - e_i\right) \\
\iff \quad & p_i\left(e_i', e_j'\right) - p_i\left(e_i, e_j'\right) \geq p_i\left(e_i', e_j\right) - p_i\left(e_i, e_j\right)
\end{aligned}
$$

because $v_i \geq 0$. Then, the objective function satisfies increasing differences if and only if $p_i\left(e_i, e_j\right)$ satisfies increasing differences.

For the differentiable case with unique interior solution, we have that $e_i^*\left(v_i, e_j\right)$ satisfies the first-order condition $v_i \frac{\partial}{\partial e_i} p_i\left(e_i, e_j\right) = 1$. Using the implicit differentiation technique we have

$$
\frac{\partial}{\partial e_j} e_i\left(v_i, e_j\right) = -\frac{\frac{\partial^2}{\partial e_j \partial e_i} p_i\left(e_i^*\left(v_i, e_j\right), e_j\right)}{\frac{\partial^2}{\partial e_i^2} p_i\left(e_i^*\left(v_i, e_j\right), e_j\right)} \geq 0
$$

□

We have that under some conditions $e_i\left(v_i, e_j\right)$ is non-decreasing in e_j. This strategic complementarity will be key for the equilibrium behaviour of the dissipation of this conflict. A stochastic order change in the distribution of v_i will generate predictable changes in the best response $e_j\left(v_j\right)$. If $e_i\left(v_i, e_j\right)$ is non-decreasing in e_j then in equilibrium both e_i and e_j will change in the same direction and we have a known change in the dissipation.

This completes our characterization of the equilibrium responses to a first and second-order stochastic change in the distribution of v_j. We summarize the results in the following Proposition.

Proposition 4. *Let $e_i^*\left(v_i, e_j\right)$ be the unique solution to agent i's problem.*

Fix a realization $\left(\hat{v}_i, \hat{v}_j\right)$. Define $d\left(F_i, \hat{v}_i, \hat{v}_j\right) = e_i^*\left(\hat{v}_i, e_j^*\left(\hat{v}_j, F_i\right)\right) + e_j^*\left(\hat{v}_j, F_i\right)$ to be the equilibrium dissipation of the conflict under distribution F_i, where $e_j^*\left(\hat{v}_j, F_i\right)$ is the best response of agent j to the optimal strategy $e_i^*\left(\hat{v}_i, e_j\right)$ under F_i.

Assume that $e_i^*\left(v_i, e_j\right)$ is non-decreasing in e_j.

- *If G_i first-order stochastically dominates F_i and $U_j\left(e_j, t; e_i\left(v_i, e_j\right)\right)$ ($U_j\left(e_j, 1-t; e_i\left(v_i, e_j\right)\right)$) satisfies the single crossing property in $\left(e_j, t\right)$, then the equilibrium dissipation under F_i is greater (less) than or equal to the dissipation under G_i. That is, $d\left(F_i, \hat{v}_i, \hat{v}_j\right) \geq d\left(G_i, \hat{v}_i, \hat{v}_j\right)$ ($d\left(F_i, \hat{v}_i, \hat{v}_j\right) \leq d\left(G_i, \hat{v}_i, \hat{v}_j\right)$).*

- *If F_i is a mean-preserving spread of G_i and $U_j\left(e_j, t; e_i\left(v_i, e_j\right)\right)$ ($U_j\left(e_j, 1-t; e_i\left(v_i, e_j\right)\right)$) satisfies the single crossing property in $\left(e_j, t\right)$, then the equilibrium dissipation under*

F_i is greater than or equal to the dissipation under G_i.

Proof. The result follows from Theorem 2. Under the stated conditions, $e_j^*(\hat{v}_j, F_i) \geq e_j^*(\hat{v}_j, G_i)$ ($e_j^*(\hat{v}_j, F_i) \leq e_j^*(\hat{v}_j, G_i)$). Since $e_i^*(\hat{v}_i, e_j)$ is non-decreasing in e_j, in the last stage of the game agent i will optimally choose a effort level that satisfies $e_i^*\left(\hat{v}_i, e_j^*(\hat{v}_j, F_i)\right) \geq e_i^*\left(\hat{v}_i, e_j^*(\hat{v}_j, G_i)\right)$ ($e_i^*\left(\hat{v}_i, e_j^*(\hat{v}_j, F_i)\right) \leq e_i^*\left(\hat{v}_i, e_j^*(\hat{v}_j, G_i)\right)$). Then the dissipation under the distribution F_i is greater (less) than or equal to the dissipation under G_i. □

Assume that agent j knows that v_i is a random variable of the form

$$v_i = V + \epsilon$$

where $E(\epsilon|V) = 0$ for all possible values of V. That is, v_i is a mean-preserving spread of V. We could interpret ϵ as some noise in the information available to agent j. If a central planner is interested in the intensity of the conflict, it could be worthwhile to pay to avoid the noise ϵ. That is, investing in better information (less volatile beliefs on v_i) leads, under some assumptions, to a reduction in the equilibrium dissipation and a reduction in the cost of conflict.

In particular, under the stated conditions, a government could find useful to get as much information as possible about terrorist threats. Also, prosecution could find worth to invest in knowing everything about the strength of the defense in a criminal trial.

4 A simultaneous game of conflict

In this section we consider the standard simultaneous contest game. Two groups are competing to obtain a prize. They make decisions in a simultaneous way. Otherwise the game is the same as before.

In the simultaneous game, a strategy for agent i is a function $e_i : [\underline{v}_i, \overline{v}_i] \to \mathbb{R}_+$ that assigns to each type a non-negative value. The same definition applies to agent j.

Given a strategy $e_j(v_j)$ of agent j, the problem faced by agent i is:

$$\max_{e_i \geq 0} \quad E_j\left[v_i p_i\left(e_i, e_j(v_j)\right) - e_i\right]$$

where the expected value is taken with respect to the distribution $F_j(v_j)$.

The solution can be characterized by the Kuhn-Tucker condition

$$\begin{aligned} v_i \frac{\partial}{\partial e_i} \int_{[\underline{v}_j, \overline{v}_j]} p_i\left(e_i^*, e_j\left(v_j\right)\right) dF_j - 1 = 0 \qquad & \text{for } e_i^* > 0 \\ v_i \frac{\partial}{\partial e_i} \int_{[\underline{v}_j, \overline{v}_j]} p_i\left(e_i^*, e_j\left(v_j\right)\right) dF_j - 1 \leq 0 \qquad & \text{if } e_i^* = 0 \end{aligned}$$

and this defines the implicit best-response function of agent i to strategy $e_j\left(v_j\right)$. Under some assumptions we can show the best-response function to be increasing and concave in v_i.

Proposition 5. *Assume the following:*

- *The optimization problem has a unique interior solution,*
- $\frac{\partial}{\partial e_i} p_i\left(e_i, e_j\right) > 0$*,*
- $\frac{\partial^2}{\partial e_i^2} p_i\left(e_i, e_j\right) < 0$*,*
- *and* $\frac{\partial^3}{\partial e_i^3} p_i\left(e_i, e_j\right) \leq 0$ *for all* e_i*,* e_j*.*

Then the best-response function to the strategy $e_j\left(v_j\right)$ *is an increasing and concave function.*

Proof. It follows from the same argument as in the proof of Proposition 1 and the use of Leibniz's rule. □

Note that we must impose stronger conditions than those required in Proposition 1. This is due to the role of uncertainty in agent's i problem. The distribution F_j now is part of the preferences of agent j and has to be taken into account when solving for the best-response function.

Let $e_i\left(v_i\right)$ be a strategy of agent i. The problem faced by agent j is analogous to that faced by agent i. Consider two distributions $F_i\left(v_i\right)$ and $G_i\left(v_i\right)$. Let $U_j\left(e_j, t; e_i\left(v_i\right)\right)$ be the parametrized objective function

$$U_j\left(e_j, t; e_i\left(v_i\right)\right) = v_j \left(t \int_{[\underline{v}_i, \overline{v}_i]} p_j\left(e_j, e_i\left(v_i\right)\right) dF_i + (1-t) \int_{[\underline{v}_i, \overline{v}_i]} p_j\left(e_j, e_i\left(v_i\right)\right) dG_i \right) - e_j$$

for $v_j \in \left[\underline{v}_j, \overline{v}_j\right]$, $e_j \in [0, v_j]$ and parameter $t \in \{0, 1\}$.

We have a result that is analogous to Proposition 2.

Proposition 6. *Fix a strategy $e_i(v_i)$ followed by agent i. For all $e'_j > e_j$, let $\Delta p_j\left(e_i; e'_j, e_j\right) \equiv p_j\left(e'_j, e_i\right) - p_j(e_j, e_i)$.*

- *Assume that $e_i(v_i)$ is a non-decreasing function and $\Delta p_j\left(e_i; e'_j, e_j\right)$ is a non-increasing (non-decreasing) function. If G_i first-order stochastically dominates F_i, then the objective function $U_j(e_j, t; e_i(v_i))$ ($U_j(e_j, 1-t; e_i(v_i))$) satisfies the single crossing property in (e_j, t).*
- *Assume that $e_i(v_i)$ is concave and $\Delta p_j\left(e_i; e'_j, e_j\right)$ is non-increasing and convex (non-decreasing and concave). If F_i is a mean-preserving spread of G_i then the objective function $U_j(e_j, t; e_i(v_i))$ ($U_j(e_j, 1-t; e_i(v_i))$) satisfies the single crossing property in (e_j, t).*

Note that in the differentiable case, the same conclusions are obtained if $\frac{\partial}{\partial e_j}\left(\frac{\partial}{\partial e_i} p_j(e_j, e_i)\right) \geq 0$ and if $\frac{\partial}{\partial e_j} p_j(e_j, e_i)$ is concave (convex) in e_i.

Proof. Let $e'_j > e_j$. A sufficient condition for the single crossing property of $U_j(e_j, t; e_i(v_i))$ is

$$\int_{[\underline{v}_i, \overline{v}_i]} \left(p_j\left(e'_j, e_i(v_i)\right) - p_j(e_j, e_i(v_i))\right) dF_i \geq \int_{[\underline{v}_i, \overline{v}_i]} \left(p_j\left(e'_j, e_i(v_i)\right) - p_j(e_j, e_i(v_i))\right) dG_i$$

If $p_j\left(e'_j, e_i(v_i)\right) - p_j(e_j, e_i(v_i)) = \Delta p_j(v_i)$ is a constant function, this inequality is satisfied trivially.

Assume now that $\Delta p_j(v_i)$ is not constant.

If G_j first-order stochastically dominates F_j, this inequality will hold if $-\Delta p_j(v_i)$ is an non-decreasing function of v_i. This is satisfied if $e_i(e_j, v_i)$ is non-decreasing in v_i and $-\Delta p_j\left(e_i; e'_j, e_j\right)$ is non-decreasing in e_i.

If F_j is a mean-preserving spread of G_j, this inequality will hold if $-\Delta p_j(v_i)$ is a non-decreasing and concave function of v_i. This is satisfied if $e_i(e_j, v_i)$ is concave in v_i and $-\Delta p_j\left(e_i; e'_j, e_j\right)$ is non-decreasing and concave in e_i. □

To apply our analysis to equilibrium quantities, we need a result analogous to Proposition 3 to be able to relate changes in $e_j(v_j)$ with changes in $e_i(v_i)$. However, in the simultaneous game this kind of result seems unlikely to hold. The reason is that $p_i(e_i, e_j) = 1 - p_j(e_j, e_i)$ and the game will not be supermodular. If for instance we impose $\frac{\partial^2}{\partial e_j \partial e_i} p_i(e_i, e_j) >$

0 (supermodularity of i's objective function in (e_i, e_j)) we are simultaneously imposing $\frac{\partial^2}{\partial e_j \partial e_i} p_j(e_j, e_i) < 0$ (submodularity of j's objective function). Then $e_i^*(v_i)$ increases if $e_j(v_j)$ is bigger, but at the same time $e_j^*(v_j)$ decreases, given that now $e_i(v_i)$ is bigger. Since the effects go in different directions, we cannot easily conclude anything about the final change in the equilibrium dissipation when some of the distributions F_i or F_j change. Then we need to impose conditions such that the direct effect of the stochastic change is larger than the indirect effects due to changes in the opponent's behaviour.

4.1 Asymmetric information

Assume that agent i has full information. That is, the realization of both v_i and v_j are known to agent i. Then the problem of agent i in the simultaneous game simplifies to choose a best response to the strategy $e_j(v_j)$:

$$\max_{e_i \geq 0} \quad v_i p_i(e_i, e_j(v_j)) - e_i$$

Since this is the same problem solved by agent i in the sequential game presented in the previous section, we know that Proposition 1 and Proposition 3 apply.

Now consider the problem solved by agent j facing strategy $e_i(v_i)$. We will first consider the interior solution to agent j's problem. The first-order condition is

$$v_j \int_{[\underline{v}_i, \overline{v}_i]} \frac{\partial}{\partial e_j} p_j(e_j, e_i(v_i)) \, dF_i = 1$$

If $e_i(v_i)$ is the equilibrium strategy $e_i^*(v_i)$ it must satisfy the Kuhn-Tucker condition for agent i, $v_i \frac{\partial}{\partial e_i} p_i(e_i^*(v_i), e_j) \leq 1$.

This condition defines $e_i(v_i, e_j)$ and we have the equilibrium identity

$$e_i^*(v_i) = e_i\left(v_i, e_j^*(v_j)\right)$$

where $e_j^*(v_j)$ is the best response of agent j to the equilibrium strategy $e_i^*(v_i)$. Then to find $e_j^*(v_j)$ we must solve the equation

$$v_j \int_{[\underline{v}_i, \overline{v}_i]} \frac{\partial}{\partial e_j} p_j\left(e_j^*, e_i\left(v_i, e_j^*\right)\right) dF_i = 1$$

Now, let G_i first-order stochastically dominate F_i. Following the reasoning behind

Proposition 6, if $\frac{\partial}{\partial e_j} p_j(e_j, e_i(v_i, e_j))$ is a non-increasing (non-decreasing) function of v_i we have

$$\int_{[\underline{v}_i, \overline{v}_i]} \frac{\partial}{\partial e_j} p_j(e_j, e_i(v_i, e_j))\, dF_i \geq (\leq) \int_{[\underline{v}_i, \overline{v}_i]} \frac{\partial}{\partial e_j} p_j(e_j, e_i(v_i, e_j))\, dG_i$$

If F_i is a mean-preserving spread of G_i, the same inequality holds if $\frac{\partial}{\partial e_j} p_j(e_j, e_i(v_i, e_j))$ is a convex (concave) function of v_i.

Let $e_j^*(v_j, F_i)$ be such that $v_j \int_{[\underline{v}_i, \overline{v}_i]} \frac{\partial}{\partial e_j} p_j\left(e_j^*(v_j, F_i), e_i\left(v_i, e_j^*(v_j, F_i)\right)\right) dF_i = 1$. Then

$$1 \geq (\leq) \int_{[\underline{v}_i, \overline{v}_i]} \frac{\partial}{\partial e_j} p_j\left(e_j^*(v_j, F_i), e_i\left(v_i, e_j^*(v_j, F_i)\right)\right) dG_i$$

and if the inequality is strict, $e_j^*(v_j, F_i)$ is not a solution to the first-order condition under G_i.

Assume that $e_i(v_i, e_j)$ is differentiable almost everywhere. Consider

$$\begin{aligned}
\frac{d}{de_j}\left(\frac{\partial}{\partial e_j} p_j(e_j, e_i(v_i, e_j))\right) &= \frac{\partial^2}{\partial e_j^2} p_j(e_j, e_i(v_i, e_j)) + \frac{\partial^2}{\partial e_i \partial e_j} p_j(e_j, e_i(v_i, e_j)) \times \frac{\partial}{\partial e_j} e_i(v_i, e_j) \\
&= \begin{cases} \frac{\partial^2}{\partial e_j^2} p_j(e_j, e_i(\cdot)) - \left(\frac{\partial^2}{\partial e_i \partial e_j} p_j(e_j, e_i(\cdot))\right)^2 \frac{1}{\frac{\partial^2}{\partial e_i^2} p_j(e_j, e_i(\cdot))} & \text{if } e_i(v_i, e_j) > 0 \\ \frac{\partial^2}{\partial e_j^2} p_j(e_j, e_i(v_i, e_j)) & \text{if } e_i(v_i, e_j) = 0 \end{cases}
\end{aligned}$$

If $\frac{\partial^2}{\partial e_j^2} p_j(e_j, e_i) < 0$ and $\frac{\partial^2}{\partial e_i^2} p_j(e_j, e_i) > 0$ then $\frac{d}{de_j}\left(\frac{\partial}{\partial e_j} p_j(e_j, e_i(v_i, e_j))\right) < 0$ and $\frac{\partial}{\partial e_j} p_j(e_j, e_i(v_i, e_j))$ is a decreasing function in e_j. Since

$$\int_{\underline{v}_i}^{\overline{v}_i} \frac{\partial}{\partial e_j} p_j\left(e_j^*(v_j, G_i), e_i\left(v_i, e_j^*(v_j, G_i)\right)\right) dG_i \geq (\leq) \int_{\underline{v}_i}^{\overline{v}_i} \frac{\partial}{\partial e_j} p_j\left(e_j^*(v_j, F_i), e_i\left(v_i, e_j^*(v_j, F_i)\right)\right) dG_i$$

then we must have

$$e_j^*(v_j, G_i) \leq (\geq) e_j^*(v_j, F_i)$$

Finally, if $e_i(v_i, e_j)$ is non-decreasing in e_j, we get

$$e_i^*(v_i, G_i) = e_i\left(v_i, e_j^*(v_j, G_i)\right) \leq (\geq) e_i\left(v_i, e_j^*(v_j, F_i)\right) = e_i^*(v_i, F_i)$$

In this simplified case of asymmetric information and interior solution, under several assumptions, we can conclude that

$$e_i^*(v_i, G_i) + e_j^*(v_j, G_i) \leq (\geq) e_i^*(v_i, F_i) + e_j^*(v_j, F_i)$$

Now we will extend our analysis to the general solution of agent j's problem. Assume that $\frac{\partial}{\partial e_j} p_j\left(e_j, e_i\left(v_i, e_j\right)\right)$ is non-increasing in e_j. Let

$$\tilde{v}_j\left(F_i\right) \equiv \lim_{e_j \rightarrow 0^+}\left[\int_{\underline{v}_i}^{\overline{v}_i} \frac{\partial}{\partial e_j} p_j\left(e_j, e_i\left(v_i, e_j\right)\right) d F_i\right]^{-1}$$

The Kuhn-Tucker conditions imply that $e_j^*\left(v_i, F_i\right)=0$ if $v_j<\tilde{v}_j\left(F_i\right)$ and $e_j^*\left(v_i, F_i\right)>0$ otherwise. Note that if $\frac{\partial}{\partial e_j} p_j\left(e_j, e_i\left(v_i, e_j\right)\right)$ is non-increasing (non-decreasing) or convex (concave) in v_i then

$$\tilde{v}_j\left(G_i\right) \geq (\leq)\, \tilde{v}_j\left(F_i\right)$$

whenever G_i first-order or second-order stochastically dominates F_i. Now it is clear that the monotone comparative statics result will hold even when the solution is not interior.

This result is summarized in the following proposition.

Proposition 7. *Let $e_i\left(v_i, e_j\right)$ be the solution to the Kuhn-Tucker condition of agent i's optimization problem. Assume that*

- $\frac{\partial}{\partial e_j} p_j\left(e_j, e_i\left(v_i, e_j\right)\right)$ *is non-increasing in e_j. In the differentiable case, this condition holds if $\frac{\partial^2}{\partial e_j^2} p_j\left(e_j, e_i\left(v_i, e_j\right)\right)<0$ and $\frac{\partial^2}{\partial e_i^2} p_j\left(e_j, e_i\left(v_i, e_j\right)\right)>0$.*

- $e_i\left(v_i, e_j\right)$ *is non-decreasing in e_j. This holds if $\frac{\frac{\partial^2}{\partial e_j \partial e_i} p_j\left(e_j, e_i\left(v_i, e_j\right)\right)}{\frac{\partial^2}{\partial e_i^2} p_i\left(e_j, e_i\left(v_i, e_j\right)\right)} \leq 0$.*

Fix a realization $\left(\hat{v}_i, \hat{v}_j\right)$. Define $d\left(F_i, \hat{v}_i, \hat{v}_j\right)=e_i\left(\hat{v}_i, e_j^*\left(\hat{v}_j, F_i\right)\right)+e_j^*\left(\hat{v}_j, F_i\right)$ to be the equilibrium dissipation of the conflict under distribution F_i, where $e_j^*\left(\hat{v}_j, F_i\right)$ satisfies

$$\begin{aligned} \hat{v}_j \int_{\left[\underline{v}_i, \overline{v}_i\right]} \frac{\partial}{\partial e_j} p_j\left(e_j^*\left(\hat{v}_j, F_i\right), e_i\left(v_i, e_j^*\left(\hat{v}_j, F_i\right)\right)\right) d F_i=1 & \qquad \text{if } \hat{v}_j \geq \tilde{v}\left(F_i\right) \\ e_j^*\left(\hat{v}_j, F_i\right)=0 & \qquad \text{otherwise} \end{aligned}$$

Assume one of the following

- $\frac{\partial}{\partial e_j} p_j\left(e_j, e_i\left(v_i, e_j\right)\right)$ *is a non-increasing (non-decreasing) function of v_i and G_i first-order stochastically dominates F_i.*

- $\frac{\partial}{\partial e_j} p_j\left(e_j, e_i\left(v_i, e_j\right)\right)$ *is a convex (concave) function of v_i when $v_i \geq \tilde{v}_i$ and F_i is a mean-preserving spread of G_i.*

Then the equilibrium dissipation under F_i is greater (less) than or equal to the dissipation under G_i. That is,

$$d\left(G_i, \hat{v}_i, \hat{v}_j\right) \leq (\geq)\, d\left(F_i, \hat{v}_i, \hat{v}_j\right)$$

Both in Proposition 4 and 7 we have two main conditions for the monotone comparative statics result: a shape condition on $\frac{d}{de_j}p_j\left(e_j, e_i\left(v_i, e_j\right)\right)$ or $\frac{\partial}{\partial e_j}p_j\left(e_j, e_i\left(v_i, e_j\right)\right)$ and a strategic complementarity condition on $e_i\left(v_i, e_j\right)$. These conditions can be tracked back to conditions on $p_j\left(\cdot\right)$.

The shape condition on $\frac{\partial}{\partial e_j}p_j\left(e_j, e_i\left(v_i, e_j\right)\right)$ makes sure that the stochastic change in F_i has a predictable effect on the best response e_j. When $\frac{\partial}{\partial e_j}p_j\left(e_j, e_i\left(v_i, e_j\right)\right)$ is non-increasing the marginal benefit of an additional unit of effort e_j is lower the higher v_i is. This, together with the fact that $\frac{\partial}{\partial e_j}p_j\left(e_j, e_i\left(v_i, e_j\right)\right)$ is non-increasing in e_j, implies that when agent i is perceived as stronger (when G_i first-order stochastically dominates F_i), an increase in e_j is necessary to compensate for the constant marginal cost.

The strategic complementarity condition makes sure that the change in F_i has predictable effects on equilibrium e_i and e_j. Note however that asking for $e_i\left(v_i, e_j\right)$ to be non-decreasing in e_j is way too strong for our purposes. In fact what we require is

$$e_i\left(v_i, e_j\right) + e_j \qquad \text{is nondecreasing in } e_j$$

Then increases in e_j directly increase dissipation, even if they cause a decrease in $e_i\left(v_i, e_j\right)$. In the differentiable case this is true (assuming $\frac{\partial^2}{\partial e_i^2}p_i\left(e_j, e_i\left(v_i, e_j\right)\right) < 0$) if

$$\frac{\partial^2}{\partial e_i^2}p_i\left(e_j, e_i\left(v_i, e_j\right)\right) \leq \frac{\partial^2}{\partial e_j \partial e_i}p_j\left(e_j, e_i\left(v_i, e_j\right)\right)$$

This "weak strategic complementarity" condition is telling us that the indirect effect on e_i due to changes in e_j has to be small enough, so the direct effect dominates in equilibrium.

Finally it is important to point out that we found conditions for when asymmetric information generates higher dissipation than full and symmetric information. To see this, note that the limit distribution

$$F_{\hat{v}}\left(v\right) = \begin{cases} 0 & \text{if } \underline{v} \leq v < \hat{v} \\ 1 & \text{if } \hat{v} \leq v \leq \overline{v} \end{cases}$$

second-order stochastically dominates any distribution with expected value $\hat{v}$. This Heaviside function (the CDF of Dirac's delta measure) represents the case when agent j fully knows v_i. Then, according to Proposition 7, we have conditions on the model for when asymmetric information generates more ex-post equilibrium dissipation than the case of full symmetric information when the realization is $\hat{v}_i = \int v_i dF_i$.

4.2 Common valuation

Another easy problem to study is the one with common valuation. Let $v = v_i = v_j$ be the common valuation drawn from the distribution $F(v)$. The problem faced by the agent i is to find a best response to the strategy e_j:

$$\max_{e_i \geq 0} \qquad \int_{[\underline{v},\overline{v}]} vdF(v)\, p_i(e_i, e_j) - e_i$$

It is clear that given e_j, the distribution $F(v)$ only affects the problem through

$$\hat{v} \equiv \int_{[\underline{v},\overline{v}]} vdF(v)$$

We assume an interior solution. Let $e_i^*(\hat{v}, e_j)$ be such that it solves the first-order condition

$$\hat{v}\frac{\partial}{\partial e_i} p_i\left(e_i^*(\hat{v}, e_j), e_j\right) = 1$$

Then we have

$$\begin{aligned}
\frac{\partial}{\partial e_j} e_i^*(\hat{v}, e_j) &= -\frac{\frac{\partial^2}{\partial e_j \partial e_i} p_i\left(e_i^*(\hat{v}, e_j), e_j\right)}{\frac{\partial^2}{\partial e_i^2} p_i\left(e_i^*(\hat{v}, e_j), e_j\right)} \\
\frac{\partial}{\partial \tilde{v}} e_i^*(\hat{v}, e_j) &= -\frac{\frac{1}{\hat{v}^2}}{\frac{\partial^2}{\partial e_i^2} p_i\left(e_i^*(\hat{v}, e_j), e_j\right)}
\end{aligned}$$

The problem faced by agent j is analogous and the solution $e_j^*(\hat{v})$ satisfies the first order condition

$$\hat{v}\frac{\partial}{\partial e_j} p_j\left(e_j^*(\hat{v}), e_i^*\left(\hat{v}, e_j^*(\hat{v})\right)\right) = 1$$

We get

$$\frac{d}{d\hat{v}}e_j^*(\hat{v}) = \frac{-\frac{1}{\hat{v}^2} - \frac{\partial^2}{\partial e_i \partial e_j}p_j\left(e_j^*(\hat{v}), e_i^*\left(\hat{v}, e_j^*(\hat{v})\right)\right) \times \frac{\partial}{\partial \hat{v}}e_i^*(\hat{v}, e_j)}{\frac{\partial^2}{\partial e_j^2}p_j\left(e_j^*(\hat{v}), e_i^*\left(\hat{v}, e_j^*(\hat{v})\right)\right) + \frac{\partial^2}{\partial e_i \partial e_j}p_j\left(e_j^*(\hat{v}), e_i^*\left(\hat{v}, e_j^*(\hat{v})\right)\right) \times \frac{\partial}{\partial e_j}e_i^*(\hat{v}, e_j)}$$

$$= \frac{-\frac{1}{\hat{v}^2} - \frac{\partial^2}{\partial e_i \partial e_j}p_j\left(e_j^*(\hat{v}), e_i^*\left(\hat{v}, e_j^*(\hat{v})\right)\right) \times \frac{\partial}{\partial \hat{v}}e_i^*(\hat{v}, e_j)}{\frac{\partial^2}{\partial e_j^2}p_j\left(e_j^*(\hat{v}), e_i^*\left(\hat{v}, e_j^*(\hat{v})\right)\right) + \frac{\left(\frac{\partial^2}{\partial e_i \partial e_j}p_i\left(e_i^*\left(\hat{v}, e_j^*(\hat{v})\right), e_j^*(\hat{v})\right)\right)^2}{\frac{\partial^2}{\partial e_i^2}p_i\left(e_i^*\left(\hat{v}, e_j^*(\hat{v})\right), e_j^*(\hat{v})\right)}}$$

where we used the fact that $p_j(\cdot) = 1 - p_i(\cdot)$. Now since $\frac{\partial^2}{\partial e_i^2}p_i\left(e_i^*\left(\hat{v}, e_j^*(\hat{v})\right), e_j^*(\hat{v})\right) < 0$ and $\frac{\partial^2}{\partial e_j^2}p_j\left(e_j^*(\hat{v}), e_i^*\left(\hat{v}, e_j^*(\hat{v})\right)\right) < 0$ we have

$$\frac{d}{d\hat{v}}e_j^*(\hat{v}) \geq 0 \iff \frac{\partial^2}{\partial e_i \partial e_j}p_i\left(e_j^*(\hat{v}), e_i^*\left(\hat{v}, e_j^*(\hat{v})\right)\right) \leq -\frac{\partial^2}{\partial e_i^2}p_i\left(e_i^*(\hat{v}, e_j), e_j\right)$$

This is the "shape" condition.

In equilibrium we must have $e_i^*(\hat{v}) = e_i^*\left(\hat{v}, e_j^*(\hat{v})\right)$. Then

$$\frac{d}{d\hat{v}}e_i^*(\hat{v}) = \frac{\partial}{\partial \hat{v}}e_i^*\left(\hat{v}, e_j^*(\hat{v})\right) + \frac{\partial}{\partial e_j}e_i^*\left(\hat{v}, e_j^*(\hat{v})\right)\frac{d}{d\hat{v}}e_j^*(\hat{v})$$

Finally the effect on equilibrium dissipation is

$$\frac{d}{d\hat{v}}d(\hat{v}) = \frac{\partial}{\partial \hat{v}}e_i^*\left(\hat{v}, e_j^*(\hat{v})\right) + \left(1 + \frac{\partial}{\partial e_j}e_i^*\left(\hat{v}, e_j^*(\hat{v})\right)\right)\frac{d}{d\hat{v}}e_j^*(\hat{v})$$

$$1 + \frac{\partial}{\partial e_j}e_i^*\left(\hat{v}, e_j^*(\hat{v})\right) = \frac{\frac{\partial^2}{\partial e_i^2}p_i\left(e_i^*(\hat{v}), e_j^*(\hat{v})\right) - \frac{\partial^2}{\partial e_j \partial e_i}p_i\left(e_i^*(\hat{v}), e_j^*(\hat{v})\right)}{\frac{\partial^2}{\partial e_i^2}p_i\left(e_i^*(\hat{v}), e_j^*(\hat{v})\right)}$$

Note that

$$1 + \frac{\partial}{\partial e_j}e_i^*\left(\hat{v}, e_j^*(\hat{v})\right) \geq 0 \iff \frac{\partial^2}{\partial e_i^2}p_i\left(e_i^*(\hat{v}), e_j^*(\hat{v})\right) \leq \frac{\partial^2}{\partial e_j \partial e_i}p_i\left(e_i^*(\hat{v}), e_j^*(\hat{v})\right)$$

and this is the "weak strategic complementarity" condition. This, of course, is the same condition we got in the discussion following Proposition 7.

Then a sufficient condition for $\frac{d}{d\tilde{v}}d\left(\hat{v}\right) \geq 0$ is

$$\left|\frac{\partial^2}{\partial e_j \partial e_i} p_i\left(e_i^*\left(\hat{v}\right), e_j^*\left(\hat{v}\right)\right)\right| \leq -\frac{\partial^2}{\partial e_i^2} p_i\left(e_i^*\left(\hat{v}\right), e_j^*\left(\hat{v}\right)\right)$$

The interpretation is straightforward. Given e_j, an increase in $\hat{v}$ increases the best response $e_i^*\left(\hat{v}, e_j\right)$. However, in equilibrium, this change in $\hat{v}$ also affects the optimal e_j. We ask the indirect effect on e_i due to changes in e_j, regardless of its direction, to be small enough, so the direct effect dominates. This is standard in non-cooperative game theory (see Roy and Sabarwal (2009)).

The result is summarized in the next proposition.

Proposition 8. *Let* $\left(e_i^*\left(\hat{v}\right), e_j^*\left(\hat{v}\right)\right)$ *be an interior equilibrium of the simultaneous game with common valuation. Then if*

$$\left|\frac{\partial^2}{\partial e_j \partial e_i} p_i\left(e_i^*\left(\hat{v}\right), e_j^*\left(\hat{v}\right)\right)\right| \leq -\frac{\partial^2}{\partial e_i^2} p_i\left(e_i^*\left(\hat{v}\right), e_j^*\left(\hat{v}\right)\right)$$

we have

$$\frac{d}{d\tilde{v}}\left(e_i^*\left(\hat{v}\right) + e_j^*\left(\hat{v}\right)\right) \geq 0$$

Therefore dissipation is non-decreasing with the expected value of v.

5 Examples

In this section we provide some examples applying the results obtained in previous sections. To do this we need to choose different contest success functions.

Classical contest success functions include the "ratio CSF"

$$p_i = \frac{\alpha_i e_i}{\alpha_i e_i + \alpha_j e_j}$$

where $\alpha_i, \alpha_j > 0$, and the "difference CSF"

$$p_i = \frac{\exp\left(\alpha_i e_i\right)}{\exp\left(\alpha_i e_i\right) + \exp\left(\alpha_j e_j\right)}$$

where $\alpha_i, \alpha_j > 0$ (see Hirshleifer (1989)).

Both fit in the more general form

$$p_i = \frac{\phi(\alpha_i e_i)}{\phi(\alpha_i e_i) + \phi(\alpha_j e_j)}$$

where $\phi(\cdot)$ is non-decreasing (see Skaperdas (1996)).

5.1 Sequential game

Note that for the ratio CSF we have

$$\begin{aligned}
e_i^*(v_i, e_j) &= \begin{cases} \frac{\sqrt{\alpha_j e_j}\left(\sqrt{\alpha_i v_i} - \sqrt{\alpha_j e_j}\right)}{\alpha_i} & \text{if } \alpha_i v_i \geq \alpha_j e_j \\ 0 & \text{otherwise} \end{cases} \\
p_j(e_j, e_i^*(v_i, e_j)) &= \begin{cases} \sqrt{\frac{\alpha_j e_j}{\alpha_i v_i}} & \text{if } \alpha_i v_i \geq \alpha_j e_j \\ 1 & \text{otherwise} \end{cases} \\
\Delta p_j(v_i) &= \begin{cases} \frac{\sqrt{\alpha_j e_j'} - \sqrt{\alpha_j e_j}}{\sqrt{\alpha_i}} v_i^{-\frac{1}{2}} & \text{if } \alpha_i v_i \geq \alpha_j e_j' \\ 1 - \frac{\sqrt{\alpha_j e_j}}{\sqrt{\alpha_i}} v_i^{-\frac{1}{2}} & \text{if } \alpha_j e_j' \geq \alpha_i v_i > \alpha_j e_j \\ 0 & \text{otherwise} \end{cases}
\end{aligned}$$

where we assumed for simplicity that $p_j(0,0) = 1$.

If $\alpha_i v_i \geq \alpha_j e_j'$ for all e_j' then the single-crossing properties of agent j's objective function are satisfied (Proposition 2).

If $\frac{\alpha_i v_i}{\alpha_j} \geq 4v_j$ we have that $\frac{\sqrt{\alpha_j e_j}\left(\sqrt{\alpha_i v_i} - \sqrt{\alpha_j e_j}\right)}{\alpha_i}$ is an increasing function of e_j. Then if the informed player is strong enough we have that first and second-order stochastic changes in the distribution of v_i increase dissipation.

For the difference CSF we have

$$
\begin{aligned}
e_i^*(v_i, e_j) &= \begin{cases} \frac{1}{\alpha_i}\left(\alpha_j e_j + 2\ln\left(\frac{\sqrt{\alpha_i v_i}+\sqrt{\alpha_i v_i - 4}}{2}\right)\right) & \text{if } \alpha_i v_i \geq 4 \\ 0 & \text{otherwise} \end{cases} \\
p_j(e_j, e_i^*(v_i, e_j)) &= \begin{cases} \left[1+\left(\frac{\sqrt{\alpha_i v_i}+\sqrt{\alpha_i v_i - 4}}{2}\right)^2\right]^{-1} & \text{if } \alpha_i v_i \geq 4 \\ \frac{\exp(\alpha_j e_j)}{1+\exp(\alpha_j e_j)} & \text{otherwise} \end{cases} \\
\Delta p_j(v_i) &\equiv \begin{cases} 0 & \text{if } \alpha_i v_i \geq 4 \\ \frac{\exp\left(\alpha_j e_j'\right)-\exp(\alpha_j e_j)}{\left(1+\exp\left(\alpha_j e_j'\right)\right)(1+\exp(\alpha_j e_j))} & \text{otherwise} \end{cases}
\end{aligned}
$$

If we assume that $\alpha_i v_i \geq 4$ for all possible v_i then $\Delta p_j(v_j) \equiv 0$ and the single crossing property of agent j's objective function holds trivially. If we assume that $\alpha_i v_i < 4$ for all possible v_i then $\Delta p_j(v_j)$ is constant and again the single crossing property holds. In those cases the problem faced by agent j has no interior solution. Stochastic changes in the distribution of v_i will not affect the equilibrium level of e_j. Then Proposition 2 applies trivially.

Note that for the difference CSF, Proposition 4 implies that dissipation is non-increasing with these stochastic changes. In this case we actually have that dissipation is always constant. If $\alpha_i v_i \geq 4$ then we have $e_j^*(v_j) \equiv 0$ and $e_i^*(v_i, v_j) = \frac{2}{\alpha_i}\ln\left(\frac{\sqrt{\alpha_i v_i}+\sqrt{\alpha_i v_i - 4}}{2}\right)$.

5.2 Asymmetric information

Note that for the ratio CSF we have

$$
\begin{aligned}
\frac{\partial}{\partial e_j} p_j(e_j, e_i(v_i, e_j)) &= \begin{cases} \frac{\alpha_j}{\sqrt{\alpha_i}}\frac{1}{\sqrt{\alpha_j e_j}}\frac{1}{\sqrt{v_i}} - \frac{\alpha_j}{\alpha_i}\frac{1}{v_i} & \text{if } \alpha_i v_i \geq \alpha_j e_j \\ 0 & \text{otherwise} \end{cases} \\
\frac{\partial^2}{\partial v_i^2}\left(\frac{\partial}{\partial e_j} p_j(e_j, e_i(v_i, e_j))\right) &= \begin{cases} \alpha_j v_i^{-2}\left(\frac{3}{4}\frac{1}{\sqrt{\alpha_i v_i}}\frac{1}{\sqrt{\alpha_j e_j}} - 2\frac{1}{\alpha_i v_i}\right) & \text{if } \alpha_i v_i \geq \alpha_j e_j \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
$$

The function $\frac{\partial}{\partial e_j} p_j(e_j, e_i(v_i, e_j))$ is non-increasing in e_j. Note that $e_i(v_i, e_j)$ is increasing in e_j if and only if $\alpha_i v_i \geq 4\alpha_j e_j$, and $\frac{\partial^2}{\partial v_i^2}\left(\frac{\partial}{\partial e_j} p_j(e_j, e_i(v_i, e_j))\right) \geq 0$ if and only if $\alpha_i v_i \geq \frac{64}{9}\alpha_j e_j$. If we demand that $\alpha_i \underline{v}_i \geq \frac{64}{9}\alpha_j \overline{v}_j$ then we will have both $e_i(v_i, e_j)$ increasing in e_j and $\frac{\partial}{\partial e_j} p_j(e_j, e_i(v_i, e_j))$ convex in v_i and the monotone comparative statics result will hold.

Intuitively, this condition implies that the informed player is strong enough with respect to the uninformed player.

For the difference CSF we have

$$\begin{aligned}\exp\left(\alpha_i e_i\left(v_i,e_j\right)\right) &= \exp\left(\alpha_j e_j\right)\left(\frac{\sqrt{\alpha_i v_i}+\sqrt{\alpha_i v_i-4}}{2}\right)^2 \\ \frac{\partial}{\partial e_j}p_j\left(e_j,e_i\left(v_i,e_j\right)\right) &= \frac{\alpha_j \exp\left(\alpha_i e_i^*\right)\exp\left(\alpha_j e_j\right)}{\left(\exp\left(\alpha_i e_i^*\right)+\exp\left(\alpha_j e_j\right)\right)^2}=\frac{\alpha_j}{\alpha_i v_i}\end{aligned}$$

where $\alpha_i v_i \geq 4$. Then $\frac{\partial}{\partial e_j}p_j\left(e_j,e_i^*\left(v_i,e_j\right)\right)$ is non-increasing in e_j and convex in v_i. For simplicity, assume that e_j is bounded above. In particular, assume that the feasibility set is $0 \leq e_j \leq v_j$. Let

$$\tilde{v}_j\left(F_i\right)=\left[\int_{\left[\underline{v}_i,\overline{v}_i\right]}\frac{\alpha_j\left(\frac{\sqrt{\alpha_i v_i}+\sqrt{\alpha_i v_i-4}}{2}\right)^2}{\left(1+\left(\frac{\sqrt{\alpha_i v_i}+\sqrt{\alpha_i v_i-4}}{2}\right)^2\right)^2}dF_i\right]^{-1}$$

We have in equilibrium

$$\begin{aligned}e_j^*\left(v_j\right) &= \begin{cases}0 & \text{if } v_j<\tilde{v}_j\left(F_i\right)\\ \left[0,v_j\right] & \text{if } v_j=\tilde{v}_j\left(F_i\right)\\ v_j & \text{if } v_j>\tilde{v}_j\left(F_i\right)\end{cases}\\ e_i^*\left(v_i,v_j\right) &= \begin{cases}2\ln\left(\frac{\sqrt{\alpha_i v_i}+\sqrt{\alpha_i v_i-4}}{2}\right) & \text{if } v_j<\tilde{v}_j\left(F_i\right)\\ \left[0,v_j\right]+2\ln\left(\frac{\sqrt{\alpha_i v_i}+\sqrt{\alpha_i v_i-4}}{2}\right) & \text{if } v_j=\tilde{v}_j\left(F_i\right)\\ v_j+2\ln\left(\frac{\sqrt{\alpha_i v_i}+\sqrt{\alpha_i v_i-4}}{2}\right) & \text{if } v_j>\tilde{v}_j\left(F_i\right)\end{cases}\end{aligned}$$

Now, $\frac{\alpha_j\left(\frac{\sqrt{\alpha_i v_i}+\sqrt{\alpha_i v_i-4}}{2}\right)^2}{\left(1+\left(\frac{\sqrt{\alpha_i v_i}+\sqrt{\alpha_i v_i-4}}{2}\right)^2\right)^2}$ is decreasing and convex. Let G_i stochastically dominate F_i. We have

$$\tilde{v}_j\left(G_i\right)\geq\tilde{v}_j\left(F_i\right)$$

From the discussion previous to Proposition 7 it is easy to see that for all $\hat{v}_i$ and $\hat{v}_j$ such that $\alpha_i \hat{v}_i \geq 4$ we have

$$d(\hat{v}_i, \hat{v}_j, F_i) \geq d(\hat{v}_i, \hat{v}_j, G_i)$$

Then the monotone comparative statics result holds.

Note that in both examples we required the informed player to be significantly strong. When perceptions of agent j change in a way such that she believes that i is somehow stronger, it is optimal for her to decrease effort, because she is struggling against a really strong opponent. Agent i knows this and in equilibrium reduces her effort as well.

5.3 Common valuation

Assume that

$$p_i(e_i, e_j) = \frac{\phi(e_i)}{\phi(e_i) + \phi(e_j)}$$

where $\phi(x)$ is non-decreasing.

The first-order condition for agent i is

$$\hat{v} \frac{\phi(e_j)\,\phi'(e_i)}{(\phi(e_i) + \phi(e_j))^2} = 1$$

and in this highly symmetric game we also have

$$\hat{v} \frac{\phi(e_i)\,\phi'(e_j)}{(\phi(e_i) + \phi(e_j))^2} = 1$$

as the first-order condition for agent j. Then assuming $\hat{v} > 0$ we have

$$\frac{\phi(e_j)}{\phi'(e_j)} = \frac{\phi(e_i)}{\phi'(e_i)}$$

in equilibrium. Following Wärneryd (2003) we assume that $\phi(x)$ is convex and that the function $\Phi(x) \equiv \frac{\phi(x)}{\phi'(x)}$ is monotone. Thus

$$e_i^*(\hat{v}) = e_j^*(\hat{v})$$

Therefore $\phi\left(e_i^*\left(\hat{v}\right)\right)=\phi\left(e_j^*\left(\hat{v}\right)\right)$,

$$\begin{aligned}
\frac{\partial^2}{\partial e_j \partial e_i} p_i\left(e_j^*\left(\hat{v}\right), e_i^*\left(\hat{v}\right)\right) &= \frac{\phi'\left(e_j^*\left(\hat{v}\right)\right)\phi'\left(e_i^*\left(\hat{v}\right)\right)\left(\phi\left(e_i^*\left(\hat{v}\right)\right)-\phi\left(e_j^*\left(\hat{v}\right)\right)\right)}{\left(\phi\left(e_i^*\left(\hat{v}\right)\right)+\phi\left(e_j^*\left(\hat{v}\right)\right)\right)^3}=0 \\
\frac{\partial^2}{\partial e_i^2} p_i\left(e_j^*\left(\hat{v}\right), e_i^*\left(\hat{v}\right)\right) &= \frac{\left(\phi\left(e_i\left(\hat{v}\right)\right)+\phi\left(e_j\left(\hat{v}\right)\right)\right)\phi\left(e_j\left(\hat{v}\right)\right)\phi''\left(e_i\left(\hat{v}\right)\right)-2\left(\phi'\left(e_i\left(\hat{v}\right)\right)\right)^2}{\left(\phi\left(e_i\left(\hat{v}\right)\right)+\phi\left(e_j\left(\hat{v}\right)\right)\right)^3}\leq 0
\end{aligned}$$

and the condition $\left|\frac{\partial^2}{\partial e_j \partial e_i} p_i\left(e_i^*\left(\hat{v}\right), e_j^*\left(\hat{v}\right)\right)\right| \leq -\frac{\partial^2}{\partial e_i^2} p_i\left(e_i^*\left(\hat{v}\right), e_j^*\left(\hat{v}\right)\right)$ holds since $\phi\left(x\right)$ is convex. We have that for this simple conflict with a common and unknown value of the good in dispute, dissipation is increasing with the expected value of v.

5.4 Simultaneous game with Bernoulli distributions

We analyze the discrete case with only two types for each player. We assume that

$$v_k = \begin{cases} \overline{v}_k & \text{with probability } \theta_k \\ \underline{v}_k & \text{with probability } 1-\theta_k \end{cases}$$

where $0<\underline{v}_k<\overline{v}_k<\infty$ and $k\in\{i,j\}$. The technology of the conflict is given by the ratio CSF. We follow Fey (2008) and assume interior solution.

The Kuhn-Tucker condition for agent j is

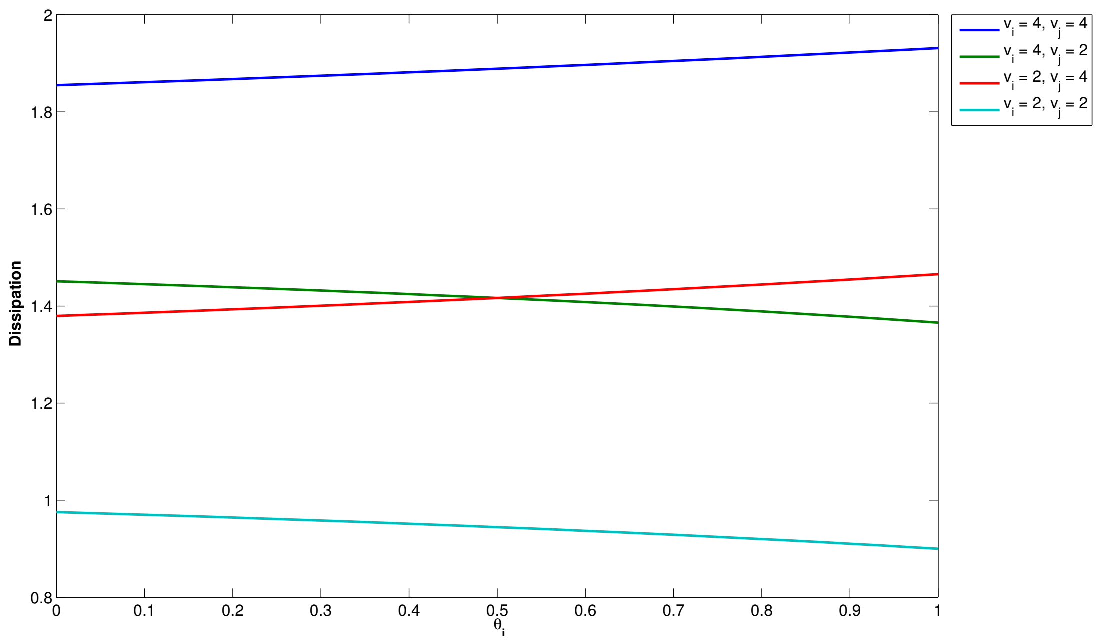
$$v_j\left(\theta_i \frac{\alpha_i\alpha_j e_i^*\left(\overline{v}_i\right)}{\left(\alpha_i e_i^*\left(\overline{v}_i\right)+\alpha_j e_j\right)^2}+\left(1-\theta_i\right)\frac{\alpha_i\alpha_j e_i^*\left(\underline{v}_i\right)}{\left(\alpha_i e_i^*\left(\underline{v}_i\right)+\alpha_j e_j\right)^2}\right)\leq 1$$

with equality in the case of interior solution. We have an analogous Kuhn-Tucker condition for agent i.

In an interior solution therefore we get four equations with four unknowns. Solving this system of equations for $e_i^*\left(\cdot\right)$ and $e_j^*\left(\cdot\right)$ give us the unique interior equilibrium.

We present a numerical example. Let $\alpha_i=\alpha_j=1$, $\underline{v}_i=\underline{v}_j=2$, $\overline{v}_i=\overline{v}_j=4$, $\theta_j=0.50$. The only parameter left is θ_i. It is easy to see that if $\theta_i'>\theta_i$ then the Bernoulli distribution with parameter θ_i' first-order stochastically dominates the Bernoulli distribution with parameter θ_i. Therefore increases in θ_i represent decreases in the stochastic order.

Figure 1: Dissipation with Bernoulli distributions



We want to analyze what happens to the interior equilibrium dissipation with changes in θ_i. The results are summarized in Figure 1.

We see that in the simultaneous game a first-stochastic change in the distribution of v_i can have different effects on the equilibrium dissipation, depending on the particular realization of (v_i, v_j).

- For high values of v_j (given that θ_j is fixed) an increase in θ_i increases the effort of agent j and decreases effort of agent i, increasing conflict's dissipation. In this case agent j believes that agent i is getting stronger. Following the reasoning of Wittman (1979), this makes the strong party more aggressive. However agent i knows this and optimally chooses to reduce her effort.

- For low values of v_j an increase in θ_i decreases the effort of both agents. Agent j believes that agent i is getting stronger. Since j is weak, her response is to reduce effort because it will be wasted in a struggle against a stronger opponent. Thus happens what Blainey (1973) argues: by knowing the real opportunities the intensity of the conflict decays.

Therefore the standard rent-seeking model is able to replicate several behaviours that seem reasonable when uncertainty changes. However, it is difficult to obtain explicit solutions and/or conditions for when the change in dissipation is predictable for all realizations (v_i, v_j).

6 Concluding remarks

We have analyzed rent-seeking contests with incomplete information. Our analysis focused on sequential contests or simultaneous games with either asymmetric information or common valuation. We found sufficient conditions for when first and second-order stochastic refinements of the prior distribution of valuations increases (decreases) the equilibrium dissipation.

For the classical contest success functions we found that if the informed agent is relatively strong with respect to the uninformed agent, then ex-post dissipation increases with second-order stochastic dominance[2]. In this case more accurate priors confirm to the uninformed player that she is really weak against the informed player. This deters her from exerting excessive effort and decreases equilibrium dissipation. Under these conditions, a central planner interested in minimizing dissipation should invest in getting more accurate information.

Much work remains to be done. The analysis can be extended for the simultaneous game when both agents have incomplete information and valuations are drawn from arbitrary distributions. This is a difficult task, since strategic complementarity for one player implies strategic substitutability for the other. The role of information in a contest with more than two players also remains to be analyzed.

[2]When the valuation is common but unknown to one agent, Wärneryd (2003) shows that under asymmetric information the ex-ante dissipation is lower than under symmetric information.

References

Blainey, G. (1973): *The Causes of War*. Free Press, New York.

Esteban, J., and D. Ray (1999): "Conflict and Distribution," *Journal of Economic Theory*, (87), 379–415.

Ewerhart, C. (2010): "Rent-seeking contests with independent private values," IEW - Working Papers iewwp490, Institute for Empirical Research in Economics - IEW.

Fey, M. (2008): "Rent-seeking contests with incomplete information," *Public Choice*, 135(3), 225–236.

Hirshleifer, J. (1989): "Conflict and Rent-Seeking Success Functions: Ratio vs. Difference Models of Relative Success," *Public Choice*, 63(2), 101–112.

——— (1993): "The Dark Side of the Force," UCLA Economics Working Papers 702, UCLA Department of Economics.

Hopkins, E., and T. Kornienko (2004): "Non-Monotone Comparative Statics in Games of Incomplete Information," ESE Discussion Papers 122, Edinburgh School of Economics, University of Edinburgh.

Malueg, D. A., and A. J. Yates (2004): "Rent Seeking with Private Values," *Public Choice*, 119(1), 161–178.

Mas-Colell, A., M. D. Whinston, and J. R. Green (1995): *Microeconomic Theory*. Oxford University Press.

Milgrom, P., and C. Shannon (1994): "Monotone Comparative Statics," *Econometrica*, 62(1), 157–80.

Rothschild, M., and J. E. Stiglitz (1970): "Increasing risk: I. A definition," *Journal of Economic Theory*, 2(3), 225–243.

Roy, S., and T. Sabarwal (2009): "Monotone Comparative Statics for Games With Strategic Substitutes," Staff General Research Papers 31558, Iowa State University, Department of Economics.

Skaperdas, S. (1996): "Contest Success Functions," *Economic Theory*, 7(2), 283–90.

Topkis, D. M. (1968): "Ordered Optimal Decisions," Ph.D. thesis, Stanford University.

——— (1998): *Supermodularity and Complementarity.* Princeton University Press.

Tullock, G. (1980): *Toward a Theory of the Rent Seeking Society*chap. Efficient Rent Seeking, pp. 97–112. Texas A&M University Press, College Station.

Wärneryd, K. (2003): "Information in Conflicts," *Journal of Economic Theory*, 110, 121–136.

Wittman, D. (1979): "How a War Ends: A Rational Model Approach," *Journal of Conflict Resolution*, 23(4), 743–763.